Note Title 3/20/2012

Lecture 15

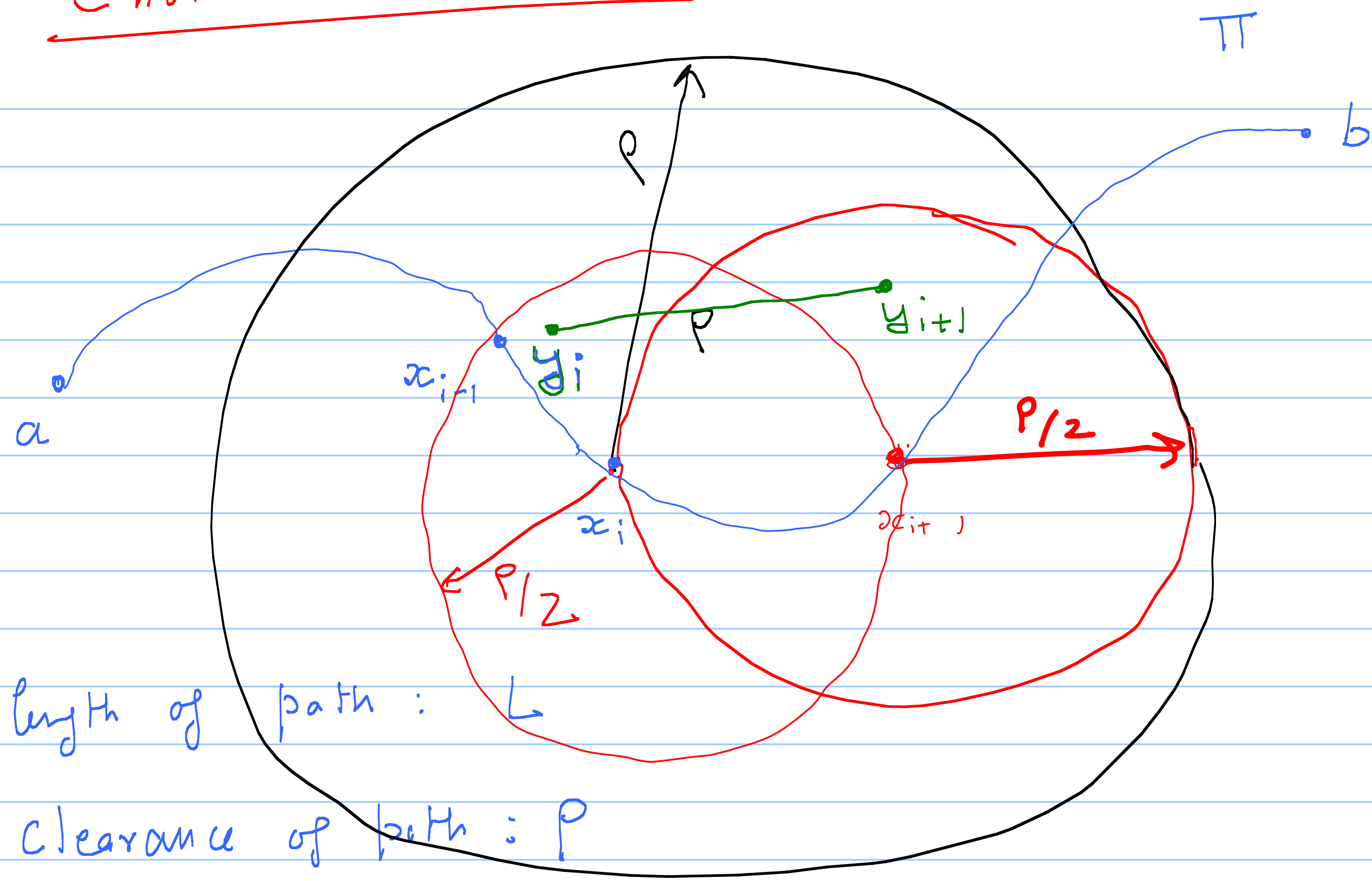
"Probabilistic Completeness" → Pr [finding a path] → 1 as $n \uparrow$

Theoretical way →

Sampling based planners have been used quite effectively in even large dim C-spaces.

→ even for dynamic obstacles (Cspace x time)

Choset's book 7.4 Section



length of path : L

Clearance of path : ρ

Local planner : st. line planner

Sample the path with discretization $\rho/2$

$m = \left\lceil \frac{L}{\rho/2} \right\rceil$ # of discret pts along the path π

by construction if (each) sample gets placed in $B_{\rho/2}(x_{i+1})$ + $B_{\rho/2}(x_i)$, then y_i + y_{i+1} are connected via an edge y_i, y_{i+1} in the roadmap,

Probability of placing a sample in $B_{\rho/2}(x_i)$:

$$= \frac{\mathrm{vol}(B_{\rho/2}(x_i))}{\mathrm{vol}(C_{free})} = \frac{\mu(B_{\rho/2}(x_i))}{\mathrm{vol}(C_{free})}$$

I_i = event such that a sample $y_i \in V$ (vertex set of roadmap) and $y_i \in B_{\rho/2}(x_i)$ is placed

$$\Pr[\text{failure}] \leq \Pr\left[\bigvee_{i=1}^{m} I_i = 0\right]$$

$$\leq \sum_{i=1}^{m} \Pr[I_i = 0]$$

$$\Pr\left[I_i = 0 \;\middle|\; \text{given } n \text{ samples in roadmap} \right]$$

$$= \left[1 - \frac{\mu\left(B_{\rho/2}(x_i)\right)}{\mu(C_{free})} \right]^n$$

$$\Pr[\text{failure}] \leq m \left[1 - \frac{\mu\left(B_{\rho/2}(x_i)\right)}{\mu(C_{free})} \right]^n \quad (*)$$

$$\sigma = \frac{\mu\left(B_1(\cdot)\right)}{2^d \mu(C_{free})}$$

$$\frac{\mu\left(B_{\rho/2}(\cdot)\right)}{\mu\left(C_{free}\right)} = \frac{\left(\frac{\rho}{2}\right)^d B_1(\cdot)}{\mu\left(C_{free}\right)} = \frac{\rho^d B_1(\cdot)}{2^d \, \mu\left(C_{free}\right)}$$

$$= \sigma \rho^d$$

and using the identity:

$$(1-\beta)^n \le e^{-\beta n}$$

from ⊛

$$\underbrace{P_r\left[\text{failure}\right]}_{r} \le \left\lceil \frac{2L}{\rho} \right\rceil \cdot e^{-\sigma \rho^d n}$$

asymptotic result.

$\ln \gamma \leq \ln\left[\frac{2L}{p}\right] - \sigma p^d n$

$\Rightarrow n \geq \ln\left[\frac{\gamma p}{2L}\right] \cdot \frac{1}{\sigma p^d}$ ← exponential in d irresp. of narrow passage being narrow in few dim

$(\varepsilon, \alpha, \beta)$ expansiveness :

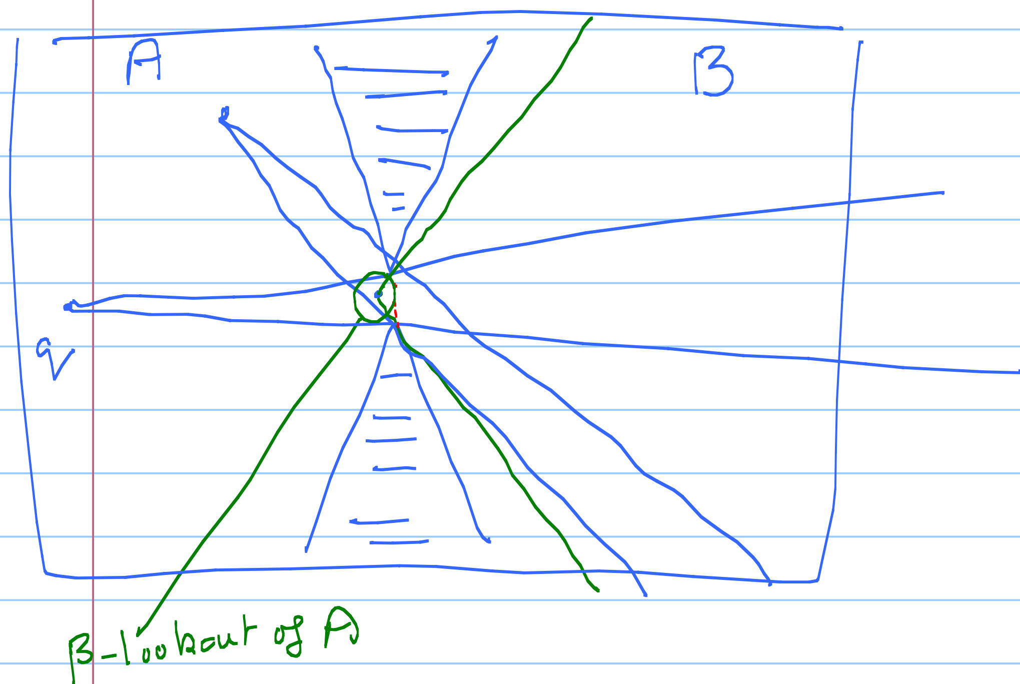
① ε-goodness : local planner → st line local planner visibility; → more gen. local planner reachability

$q \in C_{free}$

$\text{reach}(q) = \{ q' : \text{local planner}(q, q') = \text{true} \}$

$\mu(\text{reach}(q))$: vol. of reach(q)

$$\mu(\text{reach}(q)) \geq \varepsilon\, \mu(C_{free})$$



2) β-lookout of a set $S \subseteq C_{free}$: subset of S itself that can see a large fraction of $C_{free} \setminus S$ $\geq \beta\, \mu(C_{free} \setminus S)$

$$\text{lookout}_\beta(S) = \left\{ q \in S \;\middle|\; \mu(\text{reach}(q) \setminus S) \geq \beta \, \mu(C_{free} \setminus S) \right\}$$

α : size of β-lookout

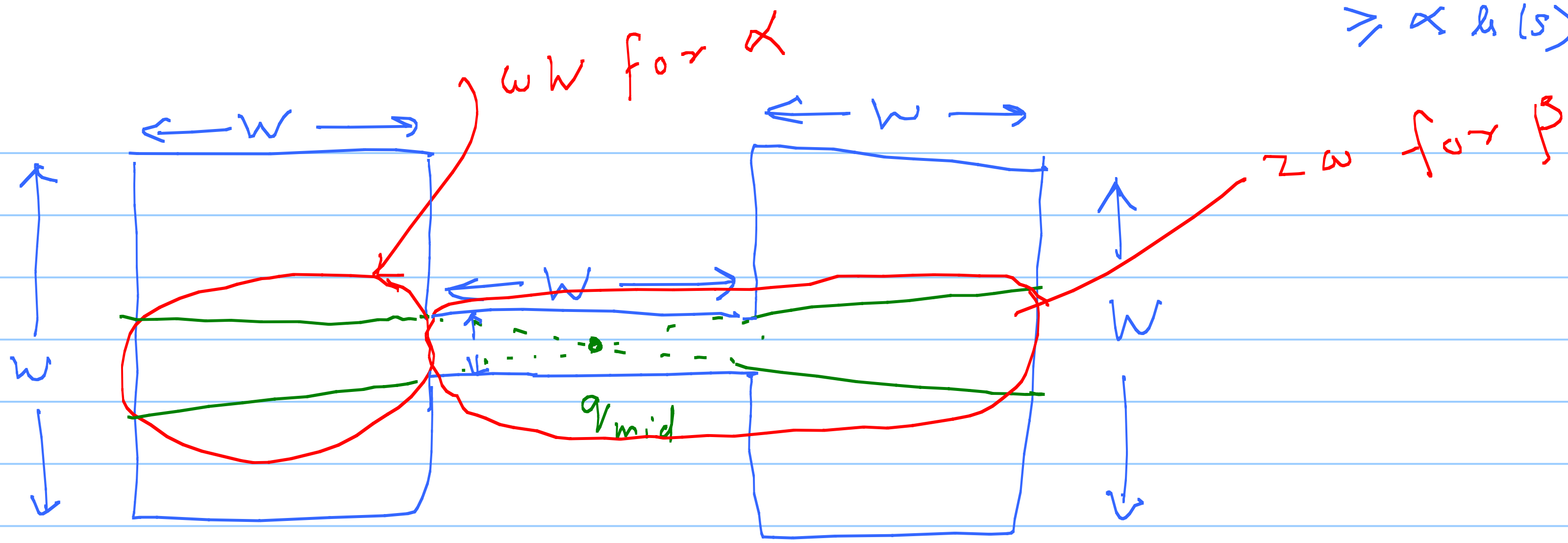
$$\mu\left[\text{lookout}_\beta(S)\right] \geq \alpha \, \mu(S)$$

Let $\alpha, \beta, \varepsilon$ be three parameters as above.

free space C_{free} is $(\alpha, \beta, \varepsilon)$ expansive if each of its component $C_{free(i)}$ satisfies:

① for every $x \in C_{free(i)}$ $\mu(\text{reach}(x)) \geq \varepsilon \, \mu(C_{free})$

② for any connected subset $S \subseteq C_{free}(i)$: $\mu(\text{lookout}_\beta(S)) \geq \alpha\, \mu(S)$



$\mu(C_{free}) = W^2 + \omega \cdot W + W^2 \approx 2W^2$

|| $\varepsilon, \alpha, \beta \rightarrow$??

$\text{reach}(q_{mid}) \approx 3\omega W$ ← min. vol. reachable set.

$\varepsilon \approx \frac{3\omega}{2W} \approx \frac{\omega}{W}$

β : Consider narrow passage as set S → $\alpha \approx 1$

$\beta = \frac{2 \cdot \omega \cdot W}{2W^2} \approx \omega / W$

if S = lgt Square o/o

$$\beta \approx \frac{2\omega W}{W^2} \approx \frac{2\omega}{W}$$

$$\alpha = \frac{2\omega W}{W^2} \approx \frac{2\omega}{W}$$

$$\approx \frac{\omega}{W}$$

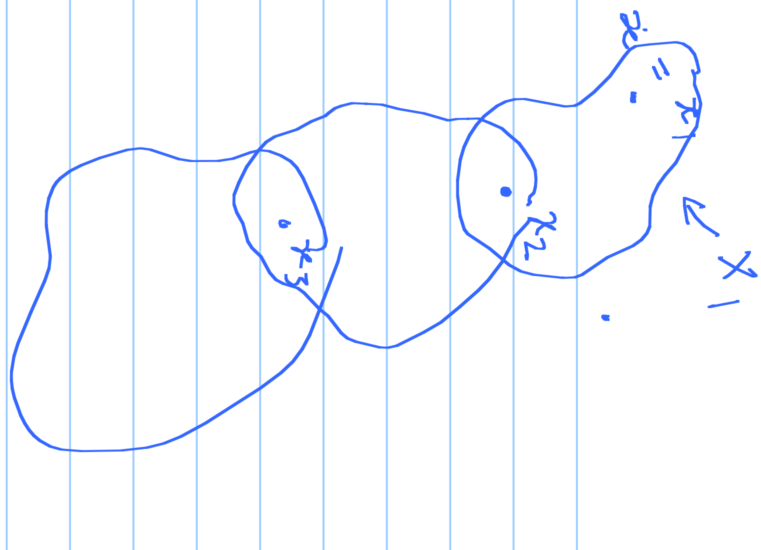
Think of S as the union of reachable sets of points in V.

Linking sequence:

$x_1 = x, x_2, \ldots x_\ell$

$X_1 = \text{reach}(x_1)$

$x_i \in \text{lookout}_\beta(X_{i-1})$ $X_i = X_{i-1} \cup \text{reach}(x_i)$

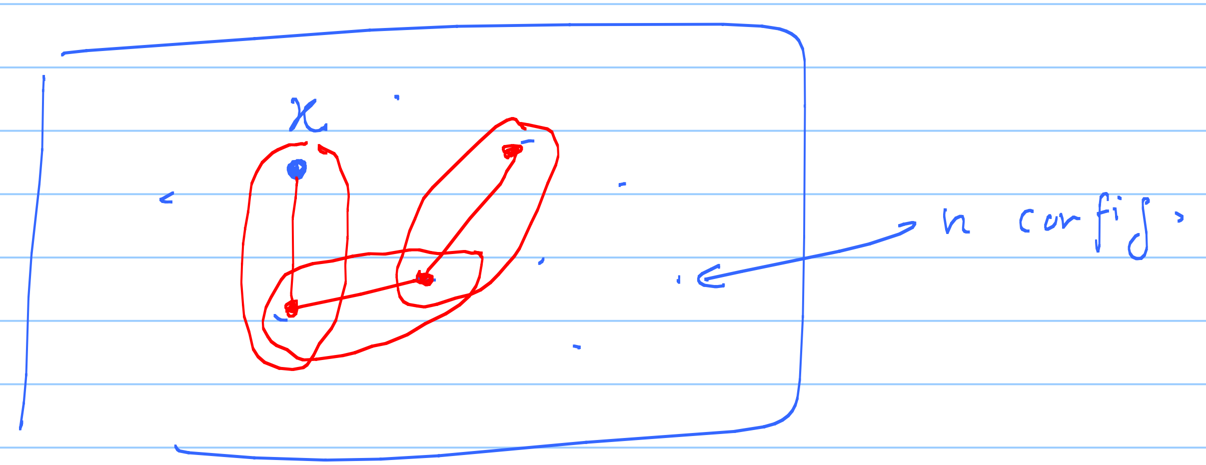


Lemma 1: Let V be a set of config. (random)

Let $s = \frac{1}{\alpha \varepsilon}$. Given any config $x \in V$

$\exists$ a linking sequence in V of ~~tg~~ length

t for x with prob. $1 - s\, e^{-(n-t-1)/s}$



For convenience $\mu(C_{free}) = 1$

L_i be the event $\exists$ a linking sequence of length i. $\bar{L}_i$

$$\Pr[\bar{L}_i] = \Pr[\bar{L}_i \mid \bar{L}_{i-1}] \Pr[\bar{L}_{i-1}] + \Pr[\bar{L}_i \mid L_{i-1}] \Pr[L_{i-1}]$$

$$\leq \Pr[\bar{L}_{i-1}] + \Pr[\bar{L}_i \mid L_{i-1}]$$

$$\mu(X_{i-1}) \geq \mu(X_1) \geq \varepsilon$$

$$\mu\left(\text{lookout}_\beta(X_{i-1})\right) \geq \alpha\, \mu(X_{i-1}) \geq \alpha\varepsilon = \frac{1}{\delta}$$

prob that V does not contain a point in $\text{lookout}_\beta(X_{i-1}) \leq \left(1 - \frac{1}{\delta}\right)^{n-i}$

$$\leq e^{-\frac{(n-i)}{\delta}}$$

$$(1-\beta)^n \leq e^{-\beta n}$$

identity

$$\Pr[\bar{E}_i] = \Pr[\bar{E}_{i-1}] + e^{-(n-i)/\delta}$$

recursive relation

$$\Pr[\bar{L}_t] \leq \sum_{i=1}^{t} e^{-(n-i)/\delta}$$

$$= e^{-(n-1)/\delta} \sum_{i=0}^{t-1} e^{i/\delta}$$

$$= e^{-(n-1)/\delta} \cdot \frac{e^{t/\delta} - 1}{e^{1/\delta} - 1}$$

$e^{1/\delta} - 1 \rightarrow \geq \frac{1}{\delta}$

$$\leq \delta e^{-(n-t-1)/\delta}$$

Lemma 2: Let $x_1 = x, x_2 \dots x_t$ be a linking sequence for $x \in C_{free(i)}$

$$\mu(X_t) = ? \geq \frac{3}{4}\mu(C_{free(i)})$$

for $t \geq \beta^{-1} \underbrace{\ln(4)}_{\frac{1.39}{\beta}}$

assume $\mu(C_{free(i)}) = 1$

$$X_i = X_{i-1} \cup reach(x_i)$$

$$l(X_i) = l(X_{i-1}) + l(\text{reach}(x_i) \setminus X_{i-1})$$

$$\geq l(X_{i-1}) + \beta \, \underbrace{l(C_{\text{frac}(i)} \setminus X_{i-1})}$$

$$l(C_{\text{frac}\,i}) - l(X_{i-1})$$

$$1 - l(X_{i-1})$$

$$\geq l(X_{i-1}) + \beta(1 - l(X_{i-1}))$$

$$\downarrow$$

$$(1-\beta)^i \underbrace{l(X_1)}_{>0} + \beta \sum_{j=0}^{i-1} (1-\beta)^j$$

$$(1-\beta) \leq e^{-\beta}$$

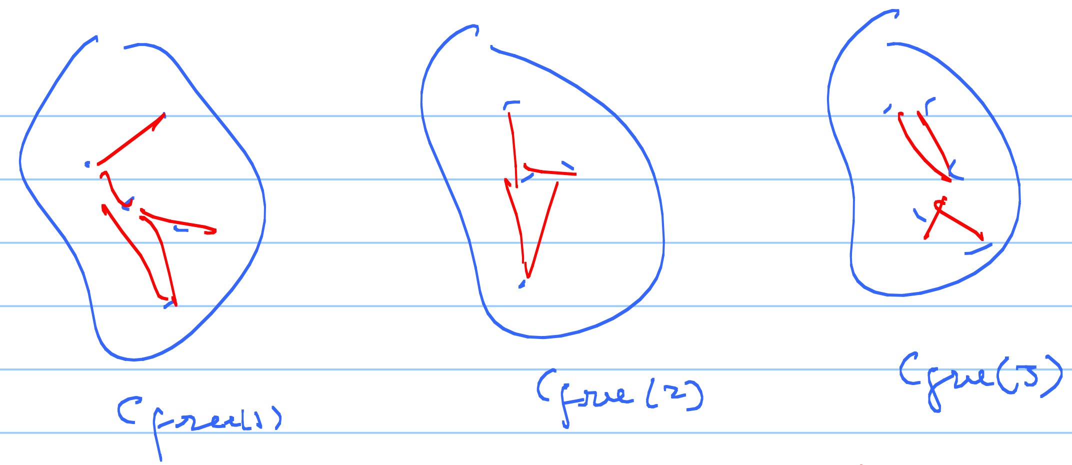
$$\Rightarrow \quad \ell(x_i) \geq 1 - e^{-\beta i}$$

for $t \geq \beta^{-1} \ln(4)$, we have $\ell(x_t) \geq \frac{3}{4}$

Theorem: Let $\delta \in [0,1]$. Suppose a set of $2n+2$ configs is chosen for

$$n = \underbrace{\frac{8 \ln\left(\frac{8}{\varepsilon \alpha \delta}\right)}{\varepsilon \alpha} + \frac{3}{\beta}}$$

randomly from C_{free}. Then with prob. at least $1-\delta$, each subgraph G_i of G_{free} is connected



Problem: estimating $\alpha, \beta, \varepsilon$ is akin to solving path planning problem

→ analytic soln. not possible except in simple cases

| still gives an explanation for success of sampling based planners.